

# A LOW FREQUENCY OSCILLATOR USING A GYRATOR

*by*

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## Abstract

This paper reports the method to obtain both the negative resistance and the simulated inductance from a single gyrator at the same time, and proposes a simple application of this technique to the realization of an oscillator. This oscillation circuit is hopeful in very low frequency range. The conditions for oscillation are given and the amplitude and frequency limitations for the circuit are analyzed.

## 1. Introduction

Recently, realizations of the functional elements which consist of the operational amplifiers and some passive elements have been widely investigated. The gyrator is one of them. The gyrator circuit proposed by Antoniou<sup>1)</sup> is well known. It consists of two operational amplifiers and four resistors, and has the advantage of simulating an inductance with a high Q-factor. There is an application of the simulated inductance obtained by a gyrator circuit terminated by a capacitor to a sinusoidal oscillation circuit. This paper reports a method to obtain both a negative resistance and a simulated inductance using a single gyrator at the same time, and proposes a new oscillator circuit using it.

## 2. Analysis

The L-C parallel resonance type oscillation circuit with a negative resistance is shown in Fig.1(a). This circuit is realized by the circuit shown in Fig.1(b), where the inductance  $L$  is simulated by the gyrator which is terminated by a capacitor, and the negative resistance  $-r$  is replaced by the negative immittance converter terminated by a positive resistor.

Through the nullor representation, it can be immediately understood that Antoniou's gyrator circuit is split into the current inversion type negative immittance converter (C.N.I.C.) and negative immittance inverter (N.I.I.). Thus, from Fig.1(b), the detailed block diagram is derived and shown in Fig.2(a).

For the L-C parallel resonance type oscillator, the stable oscillation is obtained by using the short circuit stable negative resistance. Therefore, the N.I.C. shown in Fig.1(b) should be the C.N.I.C.. For simplification, we assume that the conversion factors of two N.I.C.s in Fig.2(a) are both equal to  $-1$ . In Fig.2(a), the admittance of the resonant circuit is invariable

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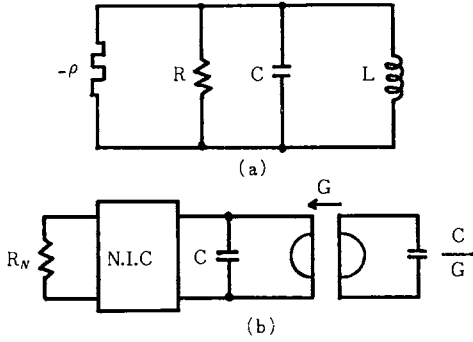


Fig. 1 Oscillation circuit with negative resistance

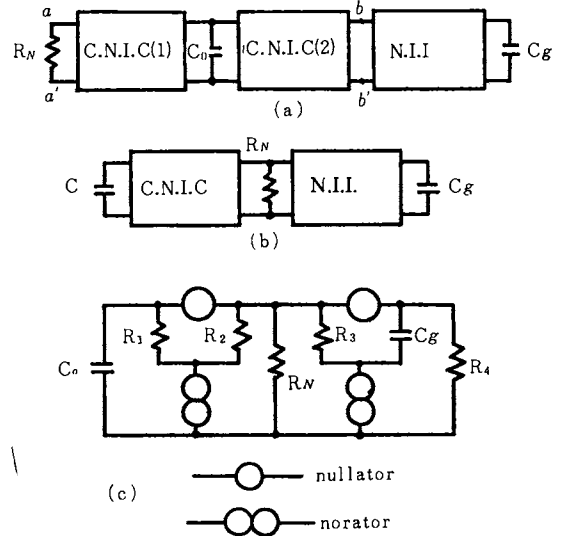


Fig. 2 (a) Block-diagram of Fig. 1 (b), (b) reduced block-diagram, (c) Nullor representation of (b).

whether the positive resistance  $R_n$  is connected to the terminals a-a' or the terminals b-b'. Thus, C.N.I.C. (1) is meaningless. In this way, simplification leads the block diagram illustrated in Fig.2(b). The nullor representation of Fig.2(b) is given in Fig.2(c). The C.N.I.C. and N.I.I. can be each constructed by an operational amplifier and two resistors. An oscillation circuit derived from nullor model is shown in Fig.3. All resistors but  $R_n$  are gyration resistors. The amplification of an operational amplifier,  $A$ , is assumed to be unipole and is given by

$$A = \frac{A_o \omega_c}{\omega_c + j\omega} \quad (1)$$

where  $A_o$  is the dc gain and  $\omega_c$  is the cutoff frequency of the amplifier.

The input admittance of the gyrator with a negative resistance is given by

$$Y_g = R_2 \left[ \frac{1}{\omega C_g R_1 R_3 R_4 Q_g} + \frac{1}{\omega C_g R_1 R_3 R_4 Q_k} - \frac{1}{R_1 R_n} + \frac{1}{j\omega C_g R_1 R_3 R_4} \right] \quad (2)$$

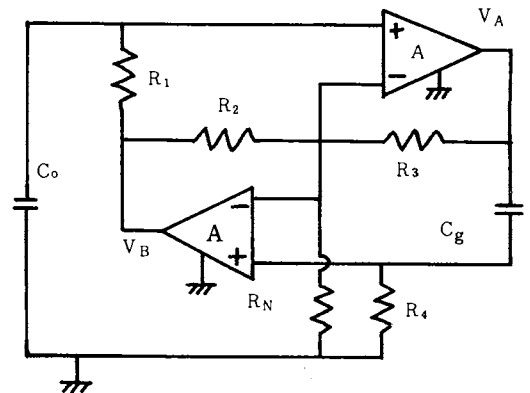


Fig. 3 Oscillator circuit

where  $Q_g$  is the Q-factor of  $C_g$ , and  $Q_k$  is the Q-factor of the gyrator terminated by an ideal capacitor and is given by

$$Q_k = \frac{1}{A_o} \left( \frac{1}{R_2} + \frac{1}{R_3} \right) \left\{ (R_3 - R_2) \frac{\omega}{\omega_c} + (\omega C_g R_3 R_4 + \frac{R_2}{\omega C_g R_4}) \right\} . \quad (3)$$

When we assume the Q-factor of an actual capacitor  $C_o$  is  $Q_o$ , the admittance of the circuit which consists of the simulated-L and  $C_o$  in parallel is given by the equation

$$Y_r = Y_g + \frac{\omega C_o}{Q_o} + j\omega C_o . \quad (4)$$

The compound Q-factor of  $Q_g$  and  $Q_o$ ,  $Q_c$ , is given as follows ;

$$Q_c = \left( \frac{1}{Q_g} + \frac{1}{Q_o} \right)^{-1} . \quad (5)$$

From the equations (2), (3) and (5),  $Y_r$  becomes

$$Y_r = \left[ \frac{1}{\omega L_g} \left( \frac{1}{Q_k} + \frac{1}{Q_c} \right) - \frac{R_2}{R_1 R_n} \right] + j \left[ \frac{\omega C_o}{1 + Q_o^{-2}} - \frac{1}{\omega L_g} \right] , \quad (6)$$

where  $L_g$  is the simulated inductance obtained by the gyrator terminated by a capacitor. The value of  $L_g$  is given by

$$L_g = \frac{R_1 R_3 R_4}{R_2} C_g . \quad (7)$$

If the loss of  $C_o$  is negligibly small and the Q-factor  $Q_o$  is much greater than unity, the admittance  $Y_r$  can be reduced to

$$Y_r \simeq \left[ \frac{1}{\omega L_g} \left( \frac{1}{Q_k} + \frac{1}{Q_c} \right) - \frac{R_2}{R_1 R_n} \right] + j \left[ \omega C_o - \frac{1}{\omega L_g} \right] . \quad (8)$$

### 3. Conditions for oscillation

The conditions for oscillation are given by

$$\left. \begin{array}{l} \text{Real} [Y_r] = 0 \\ \text{Imag} [Y_r] = 0 \end{array} \right\} . \quad (9)$$

From the equations (8) and (9), the conditions for oscillation of the circuit shown in Fig.3 are as follows ;

$$\left. \begin{array}{l} R_n = \frac{R_2}{R_1} \sqrt{\frac{L_g}{C_o} \left( \frac{1}{Q_k} + \frac{1}{Q_c} \right)^{-1}} , \\ \omega_o = 1/\sqrt{L_g C_o} . \end{array} \right\} \quad (10)$$

The equations (7) and (10) show that the oscillation frequency  $\omega_o$  is controllable by varying the gyration resistances  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$ .

By letting  $R_1 = R_2$  in the equation (10),  $R_n$  can be reduced to

$$R_n = \omega_o L_g \left( \frac{1}{Q_k} + \frac{1}{Q_c} \right)^{-1}. \quad (11)$$

The value of  $R_n$  for oscillation is shown to be dependent on the oscillation frequency. In our experiment, polyester film capacitors were used for  $C_g$  and  $C_o$  whose capacitances were equal. Their Q-factors were proved by measurement to have relations with the frequency  $\omega$  such that

$$Q_g, Q_o \propto \frac{1}{\omega^\tau} \quad (\tau < 1). \quad (12)$$

The results of measurement<sup>2)</sup> are shown in Fig.4 and Table 1.  $\tau$  lies in the range 0.23–0.27. The Q-factors at 1 KHz are

$$Q_g = Q_o = 250 \quad (1 \text{ KHz}). \quad (13)$$

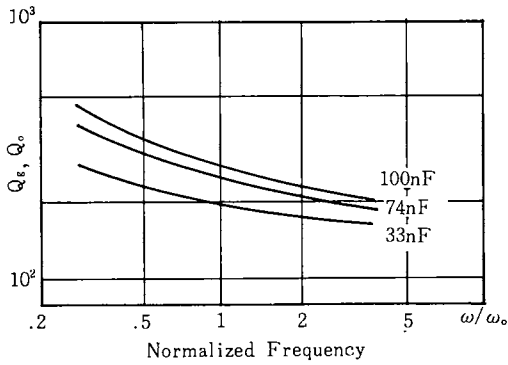


Fig. 4 Measured frequency responses of capacitors' Q-factor.

Table 1

Capacitance(nF)	$\omega_o$ (rad/s)	$\tau$ ( $\omega = \omega_o$ )
100	4902	0.27
74	6411	0.27
33	13750	0.23

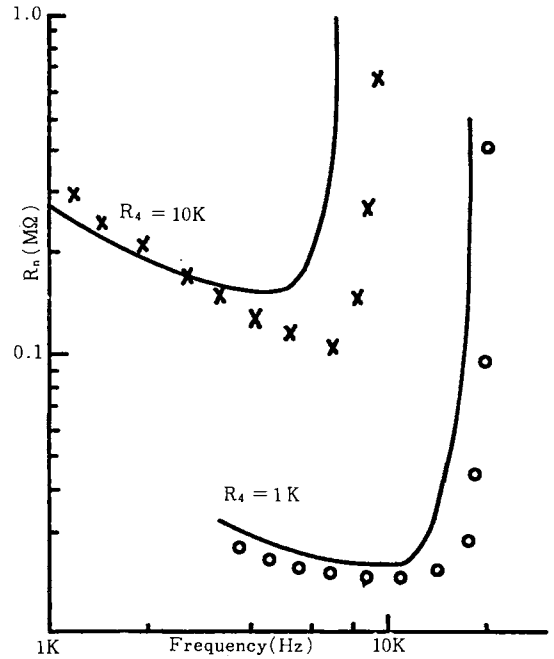


Fig. 5 Frequency responses of  $R_n$   
(calculated values; —, Measured values;  $\times \times \times$ ,  $\circ \circ \circ$ )

Substituting the expressions (12) and (13) into (11), the value of  $R_n$  for oscillation is calculated. The calculated values are shown in Fig.5, where  $A_o = 1.5 \times 10^5$  and  $\omega_c = 2\pi \times 4$  rad/sec. The experimental values are also shown in the figure.

From the equation (3), it is found that  $Q_k$  is the function of the frequency and can be made to be negative by appropriately selecting the values of  $R_2$  and  $R_3$ . However, it is

undesirable for the oscillation circuit to choose this method. Because values of the negative resistance obtained in this way are uncontrollable.

#### 4. Amplitude of oscillation

The output voltages of two operational amplifiers in Fig.3 are  $V_A$  and  $V_B$  respectively. We assume that the input-output characteristic is symmetric for the origin, and maximum output levels of  $V_A$  and  $V_B$  are the same and let them be  $V_s$ .

The relation between  $V_A$  and  $V_B$  is given by

$$V_A = F \cdot V_B \quad , \quad (14)$$

where

$$F = R_2 \left[ \frac{R_4}{Z + R_4} \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_n} \right) - \frac{1}{R_3} \right] \quad . \quad (15)$$

$Z$  is the impedance of  $C_o$  or  $C_g$  at the frequency  $\omega$  and an approximation was made that  $A_o \gg 1$ . Let maximum voltage amplitude (peak to zero) across the resonant circuit be  $V_r$ . The value of  $V_r$  is calculated and is given as follows;

$$\left. \begin{aligned} V_r &= \frac{Z}{Z + R_1} V_s & : |F| > 1 \\ V_r &= \frac{Z}{Z + R_1} F V_s & : |F| < 1 \end{aligned} \right\} \quad (16)$$

In the equation (16), the upper relation holds in the case that  $V_B = V_s$ , the lower holds in the case that  $V_A = V_s$  or in the case that  $V_A = V_B = V_s$ . If the supply voltages to the amplifiers are  $\pm 15$  volts, the maximum output voltage is about 14 volts, that is,  $V_s \simeq 14$  volts. From the equations (15) and (16),  $\bar{V}_r$  which is rms value of  $V_r$  is given by

$$\left. \begin{aligned} \bar{V}_r &= \left| \frac{Q_c + j(1 - \omega C R_1 Q_c)}{j + Q_c} \right| \frac{14}{\sqrt{2}} & : |F| > 1 \\ \bar{V}_r &= \left| \frac{Q_c + j(1 - \omega C R_1 Q_c)}{j + Q_c} \left[ \frac{j \omega C R_4 R_2}{1 + j(1 + \omega C R_4)} \left( \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_n} \right) - \frac{R_2}{R_3} \right] \right| \frac{14}{\sqrt{2}} & : |F| < 1 \end{aligned} \right\} \quad (17)$$

Fig.6 shows the output level variation with frequency. The experimental conditions are as follows;

$$\begin{aligned} R_1 &= R_2 = 15 \text{K}\Omega, R_n = 1 \text{M}\Omega, \\ C_o &= C_g = 10 \text{nF}, \\ 50 \Omega &< R_3 < 100 \text{K}\Omega, 10 \text{K}\Omega < R_4 < 500 \text{K}\Omega. \end{aligned}$$

As  $R_4$  increases, the output voltage increases and the variation of output level with frequency reduces.

We often wish for an oscillator circuit that the output level is independent of the

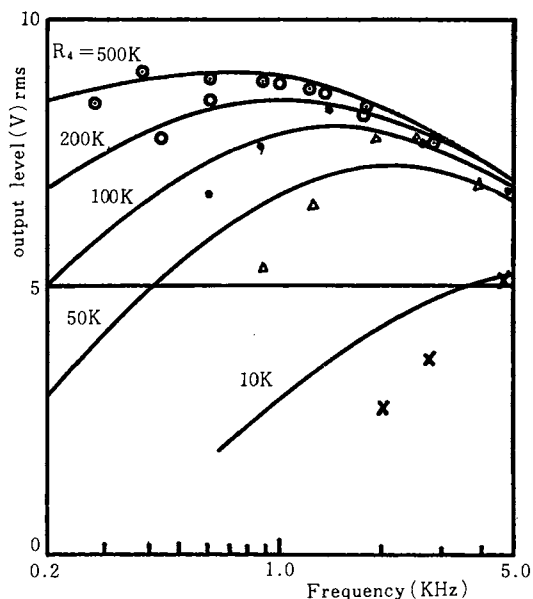


Fig. 6 Output level variation with frequency  
(calculated values; —, Measured values;  $\times \times \times$  etc.)

oscillation frequency. This demand is satisfied by connecting a diodes pair to  $C_o$  in parallel. By this method, we can obtain a constant level over a comparatively wide range for a large value of  $R_4$ .

### 5. The highest frequency

The highest oscillation frequency of the circuit is determined by the frequency limitation of the gyrator circuit. Antoniou *et al.* analyzed this type of gyrator in detail and proposed a useful model<sup>3)4)</sup>. The model for the gyrator circuit at high frequency is shown in Fig.7. In the case that  $R_1 = R_2 = R_3 = R_4 = R$ , the elements in this model are calculated as

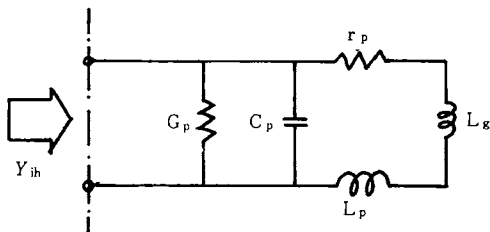


Fig. 7 Antoniou's model for gyrator circuit at high frequency

$$\left. \begin{aligned} r_p &= \frac{2\omega^2}{A_o^2 \omega_c^2} (R + 6R_o) = K\omega^2, \\ L_p &= \frac{2R + 3R_o}{A_o \omega_c}, \\ L_g &= R^2 C_g, \\ C_p &= \frac{1}{A_o \omega_c R} \left( 2 + \frac{R_o}{R} \right), \end{aligned} \right\} \quad (18)$$

where

$$K = \frac{2(R + 6R_o)}{A_o^2 \omega_c^2},$$

and  $R_o$  is the output resistance of the amplifier.

Then the driving point admittance is given by

$$\begin{aligned} Y_{ih} &= G_p + j\omega C_p + \frac{1}{r_p + j\omega L_1} \\ &= G_h + \frac{1}{j\omega X(\omega)}, \end{aligned} \quad (19)$$

where

$$X(\omega) = \frac{L_1^2 + K^2 \omega^2}{L_1 - C_p(K^2 \omega^4 + \omega^2 L_1^2)}, \quad (20)$$

$$L_1 = L_g + L_p.$$

Since the numerator of  $X(\omega)$  is always positive,  $X(\omega)$  is the inductive reactance provided the denominator is positive. However,  $X(\omega)$  is the capacitive reactance when frequency  $\omega$  is so high that the denominator is negative. Therefore, when the denominator of  $X(\omega)$  is positive, the circuit is regarded to operate as a gyrator circuit.

The frequency  $\omega_H$  at which the denominator is equal to zero is given by

$$\omega_H = \sqrt{\frac{-C_p L_1^2 + \sqrt{C_p^2 L_1^4 + 4C_p L_1 K^2}}{2C_p K^2}} \quad (21)$$

$\omega_H$  is the highest frequency that the circuit operates as a gyrator.

On the other hand, the digital computer simulation was made for this type of gyrator. The conditions employed in the simulation are as follows;

$$\begin{aligned} R_o &= 75\Omega, & A_o &= 1.5 \times 10^5, \\ \omega_c &= 2\pi \times 4 \text{ rad/sec}, & R_i &= 2\text{M}\Omega \text{ (input resistance)}, \\ R &= 1\text{K}\Omega, & C_g &= 0.1\mu\text{F}, & Q_c &= 10^5. \end{aligned}$$

The computed Q-factors of the circuit are shown in Fig.8. The results shown in Fig.8 are considered to give the Q dependence on frequency in the case that the nonideal gyrator is

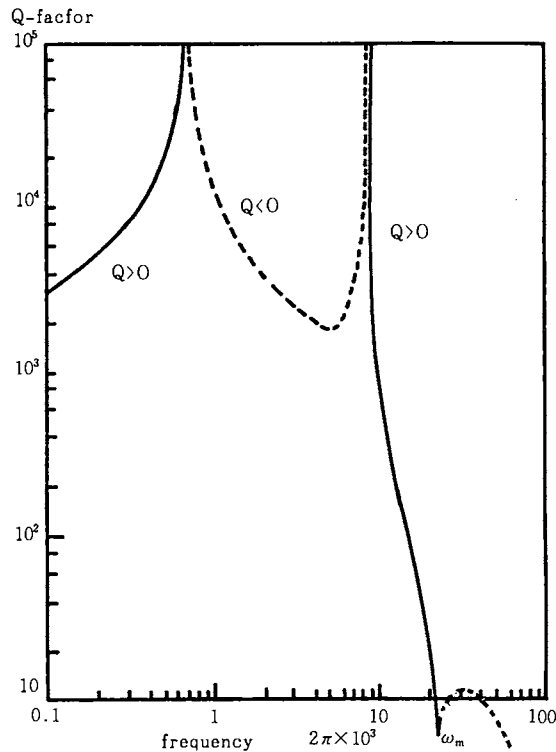


Fig. 8 Q-factor dependence on frequency  
(Dotted line indicates negative Q-factor.)

terminated by a high quality capacitor. As shown in Fig.8, Q-factor is negative above  $\omega_m$ . This means that imaginary part of the driving point impedance is negative and that the impedance is capacitive. Below  $\omega_m$ , the circuit acts as a gyrator. Hence, the highest frequency of oscillation is given by  $\omega_m$  in this analysis. The value of  $\omega_m$  computed by simulation shows good agreement with  $\omega_H$  calculated by Antoniou's model.

For the oscillation in very low frequency range ( $10^0 \sim 10^{-5}$  Hz), very large gyration resistances are needed. For example,  $R_1 = 10^6$ ,  $R_2 = 10^3$ ,  $R_3 = 10^5$ ,  $R_4 = 10^8$  in ohms for  $10^{-4}$  hertz ( $C_o = C_g = 10$  microfarads). Therefore, in order to fulfil the assumption that its input impedance should be much larger than the gyration resistances, the very high input impedance type operational amplifier is useful. We observed the sinusoidal wave at the frequency  $10^{-4}$  hertz using the MOSFET-input operational amplifiers.

## 6. Conclusions

A method of obtaining the negative resistance and simulated inductance is derived from a nullor model analysis for Antoniou's gyrator. A new realization of oscillator using this technique is proposed and its various characteristics are shown theoretically and experimentally. The variation in the loss of load capacitor with frequency was also taken into



account. The oscillation frequency is adjustable only by the gyration resistances. By using high quality capacitors with large capacitance and high input impedance amplifiers (*e.g.* MOSFET input operational amplifiers), the oscillator proposed here operates at the lower frequency than  $10^{-4}$  hertz.

### Acknowledgment

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